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Proximity effect gaps in S/N/FI structures

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Abstract. We study the proximity effect in hybrid structures consisting of superconductor and ferromagnetic insulator separated by a normal diffusive metal (S/N/FI structures). These stuctures were proposed to realize the absolute spin-valve effect. We pay special attention to the gaps in the density of states of the normal part. We show that the effect of the ferromagnet is twofold: It not only shifts the density of states but also provides suppression of the gap. The mechanism of this suppression is remarkably similar to that due to magnetic impurities. Our results are obtained from the solution of one-dimensional Usadel equation supplemented with boundary conditions for matrix current at both interfaces.

PACS. 74.45.+c Proximity effects; Andreev effect; SN and SNS junctions – 72.10.-d Theory of electronic transport; scattering mechanisms – 74.78.-w Superconducting films and low-dimensional structures – 75.70.-i Magnetic properties of thin films, surfaces, and interfaces

1 Introduction

The research on heterostructures that combine superconductors and ferromagnets has begun in sixties [1]. Still, the F/S structures remain a subject of active experimental and theoretical investigation. New developments concern Josephson, π-junctions [2], spin valves based on giant magnetoresistance effect [3], triplet superconducting ordering [4], Andreev reflection phenomena in $S-F$ trilayers [5] and proposed detection of electron entanglement [6].

Near the F/S interface electrons are influenced by both exchange field h of the ferromagnet and pair potential Δ of the superconductor. Exchange field tends to split the density of states so that the energy bands for different spin directions are shifted in energy [7]. Besides, the exchange field at F/S interfaces induces pair breaking, suppression of the spectrum gap and even formation of a superconducting gapless state [8]. The latter is qualitatively similar to the gapless state in superconductors with paramagnetic impurities [9]. The exchange field is also active if the ferromagnet is an insulator (FI): Although in this case electrons can not penetrate the ferromagnet, they pick up the exchange field while reflecting from the FI/S interface [10,11]. This has been experimentally verified with EuO-Al | Al_2O_3 | Al junctions [12].

The physics of F/S or FI/S structures is thus governed by two factors: i. electron states are modified by Δ and h, ii. the Δ and h are modified as a result of the modification of electron states by virtue of self-consistency equations. Δ and h correspond to incompatible types of ordering that suppress each other and therefore compete rather than collaborate. It was suggested [13] that $S/N/FI$ structures can be used to get rid of the second factor. The buffer normal metal effectively separates Δ and h in space to prevent their mutual suppression, provided its thickness exceeds the superconducting coherence length. However, the electrons in the normal do feel both superconducting and ferromagnetic correlations. Varying the conductances of the S/N and N/FI interfaces it is possible to tune the strength of these correlations.

If there are no ferromagnetic correlations, the traditional proximity effect in S/N structures takes place. A S/N interface couples electrons and holes in the normal metal by the coherent process of Andreev reflection [14] at energies $\varepsilon \simeq \Delta$ [15], so that Andreev bound states are formed [16]. If the normal metal is connected to the bulk superconductor only, there is a (mini) gap in the spectrum of these states. The energy scale of the gap is not Δ. Rather, it is set by the inverse escape time into the superconductor, $\hbar/\tau_E \ll \Delta$. This minigap was first predicted in reference [17] and has been intensively investigated [18]. A common realistic assumption is that diffusive transport takes place in the normal metal [19]. In this case, the superconducting proximity effect is described by the Usadel equation [20].

The $S/N/FI$ structures can be used to achieve an *absolute spin-valve effect* [13]. The collaboration of superconducting and ferromagnetic correlations results in a spin-split BCS-like DOS in the normal metal part, very much like in a BCS superconductor in the presence of the

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spin magnetic field [7]. However, the effective exchange field $\tilde{h}$ and proximity gap $\tilde{\Delta}$ characterizing the DOS in this case [13], are parametrically different from h and Δ in the ferromagnet and superconductor. In particular, the fact that the effective exchange field $\tilde{h}$ affects electrons in all points of the normal part of a $S/N/FI$ structure, does not imply to that the "real" exchange field h in the ferromagnet penetrates into N by an appreciable distance. Actually it is known that h only penetrates up to distances of the Fermi wavelength in the normal part. Rather, the effect of $\tilde{h}$ is due to the extended nature of the electron wave functions in N, which probe the spin dependent potential at the FI/N interface and carry the information about this potential throughout the whole normal metal region.

Two such stuctures with normal metal parts connected by a tunnel junction constitute the absolute spin valve [13]. The presence of a normal metal is essential to provide a physical separation between the sources of superconducting and magnetic correlations so that superconductivity and magnetism do not compete. The fact that the magnet is an insulator guarantees the absence of the normal electrons at the Fermi level and thus enables the proximity gap. The spin-valve effect mentioned would not be absolute if the ferromagnet is a metal. In this case, there would be a finite density of states at any energy due to the possible electron escape into the ferromagnet. Besides, the use of a insulator does not require nearly fully spin polarized ferromagnets (half-metal materials) to achieve an absolute spin-valve effect.

The analysis of reference [13] was restricted to the so-called "circuit-theory" limit [23]. The variation of Green functions along the normal part was disregarded. This is justified if the resistance of the normal metal part is much smaller than both the resistance of the S/N interface and effective spin-mixing resistance characterizing the N/FI interface.

In the present paper, we extend this analysis to arbitrary resistances of the diffusive normal metal part. To do so, we analyse the solutions of one-dimensional Usadel equation [20] in the normal part. Our goal is to find the gaps in the spectrum for both spin directions. The equation must be supplemented by boundary conditions at both magnetic and superconducting interfaces.

Microscopic models for interfaces of mesoscopic structures have been extensively studied in past years combining the scattering matrix approach and quasiclassical Green's function theory [21–24]. In the case of diffusive conductors the interfaces can be described in a compact/transparent way by means of "circuit theory" boundary conditions [23]. In that case the interface is described by a set of conductance parameters given as certain specific combinations of the reflection and/or transmission amplitudes of the scattering matrix associated with the interface. The regime of diffusive transport considered is distinct from the (quasi)ballistic regime assumed in many studies of F/S structures [10,11,25].

To summarize the results shortly, we have shown that the effect of magnetic correlations is twofold. Firstly, these correlations shift the BCS-like densities of states in energy, with shifts being opposite for opposite spin directions [7,11,13]. In the first approximation, the absolute value of the proximity gap is not affected by the ferromagnetic insulator. Secondly, we have also found that the magnetic correlations may suppress the gap. The gap completely dissapears at some critical values of the parameters. This is qualitatively different from reference [13] and presents the effect of the resistance of the normal metal.

The mechanism of the gap suppression appears to be surprisingly similar to that due to paramagnetic impurities [9]. At qualitative level, this has been noticed in the context of FI/S structures [10,11]. However, for $S/N/FI$ structures the analogy becomes closer: the gap closing in a certain limit (see Sect. 5) is described by equations identical to those of reference [9]. We stress that there are no magnetic impurities in our model and the quasiparticles are affected by magnetism only when they are reflected at the N/FI boundary. Effective spin-flip time thus arises from interplay of magnetic correlations and diffusive scattering in the normal metal. As we have already mentioned, the spectrum gap suppression in $S/N/FI$ structures is not accompanied by suppression of the pair potential, this is in distinction from the situation described in [9–11].

The structure of the article is as follows. In Section 2 we introduce the basic equations and define the matrix current in the diffusive normal metal. In Section 3, we specify the boundary conditions for the Usadel equation by imposing matrix current conservation at the interfaces of our $S/N/FI$ system. The resulting set of equations allows us to calculate the total Green's function $\check{G}$ at any point in the diffusive wire. Analytical solutions can be only found in two limiting cases (Sects. 4 and 5). Further on, we solve numerically the equations for general values of the parameters to obtain the general boundary in parameter space that separates gap and no-gap solutions (Sect. 6). We conclude in Section 7.

2 Matrix current and Usadel equation

Let us consider a $S/N/FI$ structure with the diffusive normal metal in the form of a slab of length L and cross-section S. This accounts both for sandwich $L \ll \sqrt{S}$ and wire $L \gg \sqrt{S}$ geometries. The Usadel equation in the normal part can be presented as

$$G_N \frac{\partial}{\partial x}\left(\check{G}(x)\frac{\partial}{\partial x}\check{G}(x)\right) = -i\,\frac{\tilde{G}_Q}{\delta}\left[\varepsilon\ \check{\tau}_3, \check{G}(x)\right] \tag{1}$$

where $G_N = \sigma\ S/L$ is the conductance associated with the diffusive metal, σ being its conductivity, $\tilde{G}_Q = e^2/\hbar$, $\delta \propto 1/SL$ is the average level spacing in the metal and ε is the energy of the quasiparticles (electrons and holes) with respect to the Fermi energy E_F. $\check{G}(x)$ is an isotropic quasiclassical Green's function in Keldysh⊗Nambu⊗Spin space, which is denoted by $(^\vee)$. In equation (1), x is the coordinate normalized to the length L, $x = 0(1)$ corresponding to the superconducting (ferromagnetic) interface.

It is convenient to define the matrix current [23] as

$$\frac{\check{I}}{G_N} = -\check{G}(x)\frac{\partial}{\partial x}\check{G}(x). \quad (2)$$

Substituting equation (2) into equation (1) we present the latter as conservation law of the matrix current:

$$\frac{\partial}{\partial x}\left(-\check{I}\right) = \check{I}_{leakage}(x) \rightarrow \check{I}_0 - \check{I}_1 = \int_0^1 dx \check{I}_{leakage}(x), \quad (3)$$

where "leakage" current $\check{I}_{leakage}(x) = -i \tilde{G}_Q \left[\varepsilon\, \check{\tau}_3/\delta, \check{G}(x)\right]$. Note that the leakage current $\check{I}_{leakage}(x)$ does not contribute to the nonequilibrium (physical) charge current, given the fact that its Keldysh component is zero. This implies that the physical (charge) current is conserved through the system, as expected. On the other hand, the diagonal components of $\check{I}_{leakage}(x)$ in Keldysh space (Retarded and Advanced components $\hat{I}^{R(A)}_{leakage}(x)$), which are proportional to the energy ε, describe decoherence between electrons and holes.

By solving the Usadel equation in the normal metal, we obtain the Green's function $\check{G}(x)$ that contains information about the equilibrium spectral properties $(\hat{G}^{R(A)}(x))$ and about the nonequilibrium transport properties $(\hat{G}^K(x))$ [26]. In this paper we concentrate on spectral properties of the diffusive wire, so from now on we restrict ourselves to the retarded block in Keldysh space. This is denoted by $(\hat{\ })$. Note that the retarded Green's function $\hat{G}^R(x) \equiv \hat{G}(x)$ is still a matrix of general structure in Nambu⊗Spin space.

If there is a single ferromagnetic element in the system, $\hat{G}(x)$ is diagonal in spin space and can separated into two blocks for spin parallel (↑) and antiparallel (↓) to the magnetization of the ferromagnet. For each spin component, $\hat{G}(x)$ can be parametrized in Nambu space as $\hat{G}(x) = \cos\theta(x)\ \hat{\tau}_3 + \sin\theta(x)\ \cos\phi(x)\ \hat{\tau}_1 + \sin\theta(x)\ \sin\phi(x)\ \hat{\tau}_2$, $\hat{\tau}_{1,2,3}$ being Pauli matrices. If there is a single superconducting reservoir attached to the normal metal, the transport properties will not depend on the absolute phase of the superconductor, so that $\phi(x)$ can be set to zero $\phi(x) = 0$. Then $\hat{G}(x)$ depends on one parameter only: $\hat{G}(x) = \cos\theta(x)\ \hat{\tau}_3 + \sin\theta(x)\ \hat{\tau}_1$. The phase (difference) $\phi(x)$ maybe important if the normal metal is connected to two or more superconducting reservoirs.

Using this parameterization for $\hat{G}(x)$, the retarded block of equations (1) and (2) transforms into a differential equation for the angle $\theta(x)$:

$$\frac{\partial^2}{\partial x^2}\theta(x) + i\ \frac{2\varepsilon}{E_T}\sin\theta(x) = 0 \quad (4)$$

and

$$\hat{I} = -i\ G_N \frac{\partial\theta(x)}{\partial x}\ \hat{\tau}_2 \quad (5)$$

where we have introduced $E_T = \hbar D/L^2 \equiv G_N\ \delta/\tilde{G}_Q$, the Thouless energy associated with the normal metal.

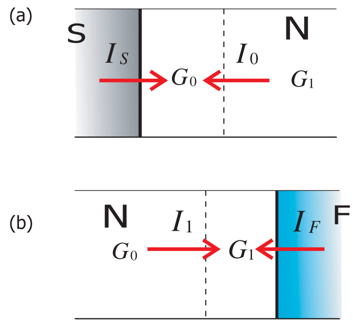

Fig. 1. Matrix current and boundary conditions. Circuit-theory expressions give matrix currents $\hat{I}_S, \hat{I}_F$ from the corresponding reservoirs. These currents should match matrix currents $\hat{I}_{0,1}$ from the Usadel equation, defined via derivatives of Green functions. This fixes the boundary conditions for the Usadel equation.

3 Boundary conditions

Circuit theory allows to find the boundary conditions at the interfaces of the normal metal in contact with the reservoirs simply by imposing matrix current conservation at these points. The matrix currents to the reservoirs are given by circuit theory expressions. (Fig. 1). We will assume that the superconducting reservoir is coupled to the normal metal through a tunnel contact. In addition, we disregard energy dependence of Green functions in the reservoir assuming that the energy scale of interest is much smaller than the superconducting energy gap Δ in the reservoir. Under these assumptions, the retarded Green function in the reservoir is just $\hat{\tau}_1$. The matrix current to the reservoir thus reads

$$\hat{I}_S = \frac{G_S}{2}\left[\hat{\tau}_1, \hat{G}(x)\right]. \quad (6)$$

To describe the matrix current to ferromagnetic insulator, we use the results of our previous work [27] where we obtain

$$\hat{I}_F = i\frac{G_\phi}{2}\left[\vec{M}\ \hat{\vec{\sigma}}\ \hat{\tau}_3, \hat{G}(x)\right], \quad (7)$$

$\vec{\sigma}$ being matrices in spin space, $\vec{M}$ being the magnetization vector of the ferromagnet.

The parameter G_ϕ has a dimension of conductance and is related to the imaginary part of so-called mixing conductance $G_\phi = \mathrm{Im}G^{\uparrow\downarrow}$. Mixing conductance has been introduced in reference [28] to describe the spin-flip of electrons reflected from a ferromagnetic boundary and is, in general, a complex number. For insulating ferromagnet it is however purely imaginary. In that case, G_ϕ acts as an effective magnetic field. Such spin-dependent scattering situation is shown to be important in magnetic insulator materials [12] and half-metallic magnets [29]. Evaluation of G_ϕ for a simple model of insulating ferromagnet can be found in reference [13].

Using the parameterization in terms of $\theta(x)$, we find

$$\hat{I}_S = -G_S\cos\theta_0\ \hat{\tau}_2 \quad (8)$$

and

$$\hat{I}_{FI} = \pm i G_\phi \sin\theta_1 \ \hat{\tau}_2, \tag{9}$$

where the $+(-)$ sign corresponds to up (down) direction of spin with respect to the magnetization axis of the ferromagnet, $\theta_0 = \theta(0)$ and $\theta_1 = \theta(1)$.

Equating $\hat{I}_S = \hat{I}_0, \hat{I}_{FI} = \hat{I}_1$ gives the boundary conditions required,

$$-g_S \cos\theta_0 = \frac{\partial\theta(x)}{\partial x}|_0 \tag{10}$$

$$\pm i g_\phi \sin\theta_1 = \frac{\partial\theta(x)}{\partial x}|_1 \tag{11}$$

where we have introduced two important dimensionless parameters characterizing the structure: $g_S = G_S/G_N$, $g_\phi = G_\phi/G_N$. As above, $\pm$ denotes two spin directions.

The solutions of the Usadel equation generally correspond to complex θ. The density of states in a given point at a given energy is $\nu(\epsilon, x) = \nu_0 \mathrm{Re}[\cos(\theta(\epsilon, x))]$, ν_0 being the density of states in the absence of proximity effect. In this paper, we concentrate on gap solutions where $\nu = 0$ everywhere in the normal metal.

For this porpuse, it is convenient to define the complex angle $\theta(x) = \pi/2 + i\ \mu(x)$. Real $\mu(x)$ corresponds to gap solution. In terms of this angle, the full system to solve reads

$$\frac{\partial^2}{\partial x^2}\mu(x) + \tilde{\varepsilon}\cosh\mu(x) = 0 \tag{12}$$

$$+g_S \sinh\mu_0 = \frac{\partial\mu(x)}{\partial x}|_0 \tag{13}$$

$$\pm g_\phi \cosh\mu_1 = \frac{\partial\mu(x)}{\partial x}|_1 \tag{14}$$

where we introduce the dimensionless energy $\tilde{\varepsilon} = 2\ \varepsilon/E_T$. The gap solutions exist in a certain region of the three-dimensional parameter space $(g_S, g_\phi, \tilde{\varepsilon})$. To determine the boundary of this region is our primary task.

There are three limits where the solutions can be obtained analytically. The limit of vanishing resistance of the normal metal, $g_S \simeq g_\phi \simeq \tilde{\varepsilon}$, can be treated with circuit theory and has been considered in reference [13]. Below we address the zero energy limit ($\tilde{\varepsilon} \approx 0$) and "spin-flip" limit ($g_S \simeq g_\phi^2 \ll 1$).

4 Zero energy

Here we will analyze the gap solutions at the Fermi level, that is, at zero energy. In this case the leakage current conveniently dissapears, $\check{I}_{leakage}(x) = 0$ and $\hat{I} = \mathrm{const}$. Equation (2) can be easily integrated giving $\hat{G}_1 = \exp(\hat{I}/G_N)\hat{G}_0$, where $\hat{G}_{0(1)} = \hat{G}(0(1))$. From this we obtain

$$\hat{I} = -\frac{G_N}{2}\frac{\arccos\left(\mathrm{Tr}\left(\hat{G}_{\mathbf{0}}\ \hat{G}_{\mathbf{1}}\right)\right)}{\sqrt{4-\left(\mathrm{Tr}\left(\hat{G}_{\mathbf{0}}\ \hat{G}_{\mathbf{1}}\right)\right)^2}}\left[\hat{G}_{\mathbf{0}}, \hat{G}_{\mathbf{1}}\right], \tag{15}$$

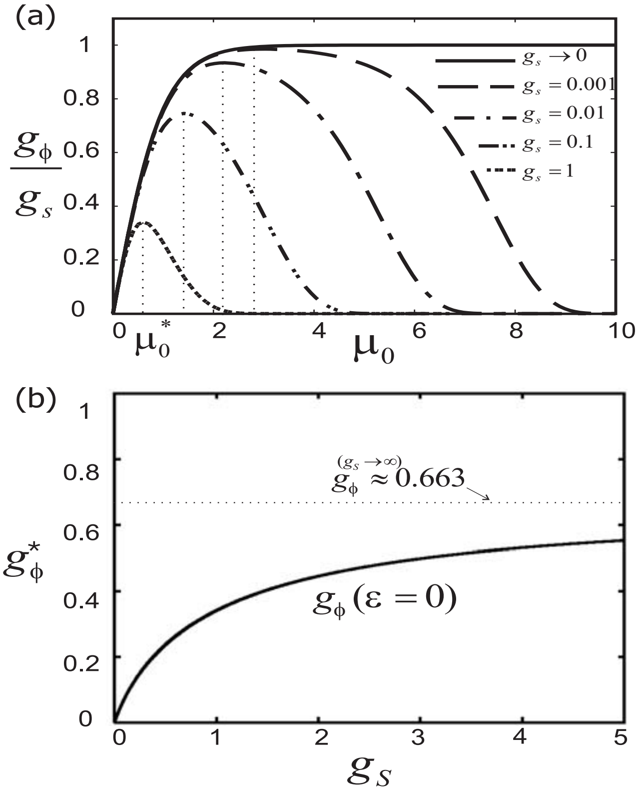

Fig. 2. (a) g_ϕ/g_S versus μ_0 for various values of g_S (Eq. (18)). The maximum achieved at μ_0^* gives the maximum g_ϕ at which the gap persists at given g_S. (b) The maximum g_ϕ^* versus g_S. The curve saturates at $g_\phi \approx 0.663$ for $g_S \to \infty$.

which can be further simplified by using the parameterization $\hat{G}_{\mathbf{0(1)}} = \cos\theta_{\mathbf{0(1)}}\ \hat{\tau}_3 + \sin\theta_{\mathbf{0(1)}}\ \hat{\tau}_1$ into the following simple expression for the current through the diffusive normal metal:

$$\hat{I} = -i\ G_N(\theta_0 - \theta_1)\hat{\tau}_2 = \ G_N(\mu_0 - \mu_1)\hat{\tau}_2. \tag{16}$$

Taking into account the boundary conditions on both interfaces, we obtain the equations for $\mu_{0,1}$,

$$g_S \sinh\mu_0 = \mu_1 - \mu_0 = \pm g_\phi \cosh\mu_1. \tag{17}$$

We can readily express from these two equations g_ϕ/g_S as a function of μ_0 and g_S

$$\frac{g_\phi}{g_S} = \frac{\sinh\mu_0}{\cosh\left(\mu_0 + g_S\sinh\mu_0\right)} = f(\mu_0,\ g_S). \tag{18}$$

Here we concentrate on the spin-up component. The solution for spin-down component corresponds to different sign of μ_0. In Figure 2a, we plot g_ϕ/g_S versus μ_0 for several values of g_S. We see that g_ϕ/g_S reaches a maximum value $(g_\phi/g_S)^*$ at a certain value μ_0^*. The position and the height of the maximum changes by changing g_S, $\mu(0)^* \to \infty, (g_\phi/g_S)^* \to 1$ if $g_S \to 0$.

So for a given g_S, the maximum possible value of g_ϕ such as there is still a gap in the induced density of states of the wire is given by

$$g_\phi^* = f(\mu_0^*, g_S)g_S. \tag{19}$$

In Figure 2b we plot g^*_ϕ as a function of g_S. This curve defines the boundary between gap (below) region and no gap (above) region in $g_\phi - g_S$ parameter space at zero energy $\varepsilon = 0$. As expected, magnetic correlations combat the proximity effect at the Fermi level and the gap solutions dissapears upon increasing g_ϕ. Even if the coupling to the superconductor is infinitely strong, $g_S \to \infty$, the gap survives only if $g_\phi < 0.663$.

Let us expand g_ϕ/g_S near its maximum value $(g_\phi/g_S)^*$. Defining deviations from this point $\Delta g_{\phi-S} = (g_\phi/g_S) - (g_\phi/g_S)^*$ and $\bar\mu = \mu_0 - \mu_0^*$, the expansion is obviously

$$\Delta g_{\phi-S} = -\,C\,\bar\mu^2 \tag{20}$$

C being a positive constant. Let us note that the density of states $\nu(0)/\nu_0 = \mathrm{Im}\ \sinh(\mu_0) \propto \mathrm{Im}\bar\mu$. This gives a square-root singularity of the density of states near the boundary,

$$\nu \propto \sqrt{\Delta g_{\phi-S}}. \tag{21}$$

The limit $g_S \to 0$ deserves some special consideration. It corresponds to the circuit-theory limit where the diffusive normal metal simply reduces to a “node” with spatially independent Green's function $\check G$. This gives however, a BCS-like *inverse* square-root singularity in the density of states at the boundary given at $g_S = g_\phi$, $\nu/\nu_0 = 1/\sqrt{1-(g_S/g_\phi)^2} \approx 1/\sqrt{2\Delta g_{\phi-S}}$ at $\Delta g_{\phi-S} \ll 1$. This result looks difficult to reconcile with the result obtained in equation (21).

The point is that for $g_S \ll 1$ an extra crossover takes place such the density of states changes from inverse square root to square root. The crossover occurs close to the gap boundary $g_S = g_\phi$. From equation (18) it is clear that for $g_S \ll 1$, the values of $g_\phi/g_S \simeq 1$ occur at $\mu_0 \gg 1$. Close to the boundary, an expansion of equation (18) up to terms $\sim \exp(-2\mu_0)$ is possible. The evaluation of the maximum of g_ϕ/g_S as function of μ_0 allows to determine the constant C in equation (20), $C \simeq g_S^{4/3}$. The crossover occurs then at $\Delta g_{\phi-S} \simeq g_S^{2/3} \ll 1$. Below the crossover, at $\Delta g_{\phi-S} \ll g_S^{2/3}$, there is a square-root singularity $\nu/\nu_0 \propto g_S^{-2/3}\sqrt{\Delta g_{\phi-S}}$ that changes to $\nu/\nu_0 \simeq 1/\sqrt{2\Delta g_{\phi-S}}$ above the crossover, at $\Delta g_{\phi-S} \gg g_S^{2/3}$. The maximum density of states is therefore $\nu/\nu_0 \simeq g_S^{-1/3}$.

5 “Spin-flip” limit

Now we would like to extend the results of the previous section to finite energy ε. We do this assuming that the Green function does not change much along the normal metal, so that $|\theta_1 - \theta_0| \ll \theta_0$.

We start again with equations (4), (10) and (11) . Integrating equation (4) over x and using equations (10) and (11) gives

$$\pm i g_\phi \sin\theta_1 + g_S\cos\theta_0 + i\ \tilde\varepsilon \int_0^1 dx \sin\theta(x) = 0. \tag{22}$$

If we assume that the Green function does not change with x, $\theta(x) = \theta_0$, we derive from equation (22) the circuit-theory equation

$$i\,(\tilde\varepsilon\ \pm g_\phi)\sin\theta_0 + g_S\cos\theta_0 = 0. \tag{23}$$

With this, we reproduce the results discussed in reference [13]: the density of states mimics the one of a BCS superconductor in the presence of the spin-splitting magnetic field

$$\upsilon(\tilde\varepsilon) = \frac{|\tilde\varepsilon \pm g_\phi|}{\sqrt{(\tilde\varepsilon \pm g_\phi)^2 - g_S^2}}. \tag{24}$$

The presence of a gap is strictly speaking a non-perturbative effect. The perturbation expansion of the density of states shown in equation (24) is valid at high energies $|\varepsilon \pm \tilde h| \gg \tilde\Delta$. The leading “spin-dependent” correction is proportional to $(\tilde\Delta^2\tilde h)/\varepsilon^3 \equiv (g_S^2 g_\phi)/\tilde\varepsilon^3$. This implies that the leading diagram in perturbation series involves four tunneling amplitudes at the S/N interface and two spin-dependent reflection amplitudes at the N/FI interface.

The equation (23) is correct in the leading order in $\tilde\varepsilon, g_S, g_\phi \ll 1$. There can be however a problem if $\tilde\varepsilon$ is too close to $\mp g_\phi$ since the first coefficient is anomalously small in this case. To account for this, we should re-derive these equations to the next-to-leading order.

We present $\theta(x)$ in the form that satisfies boundary conditions,

$$\theta(x) = \theta_0 + (\theta_1 - \theta_0)\,x + \theta^{(1)}(x);\ \theta^{(1)}(0) = \theta^{(1)}(1) = 0 \tag{25}$$

and evaluate the corrections $\theta_1 - \theta_0, \theta^{(1)}(x)$ in the leading order. To evaluate $\theta_1 - \theta_0$, let us multiply equation (4) by x and integrate by parts the first term $\int_0^1 dx\ x\ \ddot\theta(x)$ where $(\ddot\theta(x) \equiv \partial^2\theta(x)/\partial x^2)$, obtaining the following expression

$$\dot\theta_1 - (\theta_1 - \theta_0) + i\ \tilde\varepsilon \int_0^1 dx\ x\ \sin\theta(x) = 0. \tag{26}$$

We set $\theta(x) = \theta_0$ under the sign of integral and use equation (11) to obtain

$$\theta_1 - \theta_0 = i\left(\pm g_\phi\ + \frac{\tilde\varepsilon}{2}\right)\sin\theta_0. \tag{27}$$

With the same accuracy, the differential equation for $\theta^{(1)}(x)$ reads

$$\frac{\partial^2}{\partial x^2}\theta^{(1)}(x) + i\ \tilde\varepsilon\sin(\theta_0) = 0. \tag{28}$$

This results in

$$\theta^{(1)}(x) \approx i\ \tilde\varepsilon\frac{\sin\theta_0}{2}(x - x^2). \tag{29}$$

Finally we substitute equations (29) to equation (22) to get the following relation:

$$i\,(\tilde\varepsilon\ \pm g_\phi)\sin\theta_0 + g_S\cos\theta_0 - \zeta g_S\sin\theta_0\cos\theta_0 = 0, \tag{30}$$

where $\zeta = \left(g_\phi^2 \pm g_\phi\tilde{\varepsilon} + \tilde{\varepsilon}^2/3\right)/g_S$. As we mentioned above, ζ plays a role only if $|\tilde{\varepsilon} \pm g_\phi| \ll \tilde{\varepsilon}, g_\phi$. Under these conditions, the energy dependence of ζ can be disregarded and $\zeta = g_\phi^2/3g_S$.

The relation (30) resembles very much one of the most important equations in the superconductivity theory that was first derived by Abrikosov and Gor'kov [9] to describe suppression of superconductivity by magnetic impurities. Precise association is achieved by the following change of notations:

$$(\tilde{\varepsilon} \pm g_\phi)E_T \to E,$$
$$g_S E_T \to \Delta,$$
$$\zeta\Delta = E_T g_\phi^2/3 \to 1/\tau_s$$

where E, Δ, τ_s are respectively energy, superconducting order parameter and spin-flip time due to magnetic impurities [9]. This is why we refer to the limit under consideration as to "spin-flip" limit. To remind, there are no magnetic impurities in the structure considered, and effective spin-flip comes from interplay of diffusion in the normal metal and reflection at the N/FI interface.

Maki has demonstrated that similar equation accounts for gapless superconductivity in a variety of circumstances, ζ being the pair-breaking parameter [30]. We define $\theta_0 = \pi/2 + i\,\mu_0$, $u = \tanh\mu_0$, $\omega = (\tilde{\varepsilon} \pm g_\phi)/g_S$, to be close to notations of reference [30]. This gives

$$\omega = u\left(1 - \zeta\frac{1}{\sqrt{1-u^2}}\right). \tag{31}$$

The maximum value of ω with respect to real u, ω^* gives the energy interval around $\tilde{\varepsilon} = \mp g_\phi$ where the gap solutions occur. This value is determined by the condition $\partial\omega/\partial u = 0$, which gives

$$\omega^* = \left(1 - \zeta^{2/3}\right)^{3/2} \tag{32}$$

achieved at $u = u^*$,

$$u^* = \left(1 - \zeta^{2/3}\right)^{1/2}. \tag{33}$$

There are no gap solutions if $\zeta > 1$. The region where these solutions do occur is sketched in Figure 3a in $\tilde{\varepsilon} - g_\phi$ coordinates. It looks like a 45° slanted strip, the width of the stip in horizontal direction being given by $2\omega^* g_S$. Near the origin $\zeta = 0$. In this situation, it is obtained from equations (32, 33) that the width is $2g_S$ and $|\tilde{\varepsilon} \pm g_\phi| = g_S$. The gap solutions at zero energy $\tilde{\varepsilon} = 0$ dissapear at $g_\phi = g_S$. The width gradually reduces with increasing g_ϕ due to the increase of ζ. The strip ends if $\zeta = 1$, that is, at $|g_\phi| = \sqrt{3g_S} \gg g_S$ (Fig. 3b).

This demonstrates that even in the limit of $g_\phi, g_S \ll 1$ the ferromagnetic insulator not only shifts the gap states but also reduces and finally suppresses the gap due to effective spin-flip. We show in the next section that the same picture is qualitatively valid for arbitrary values of these parameters.

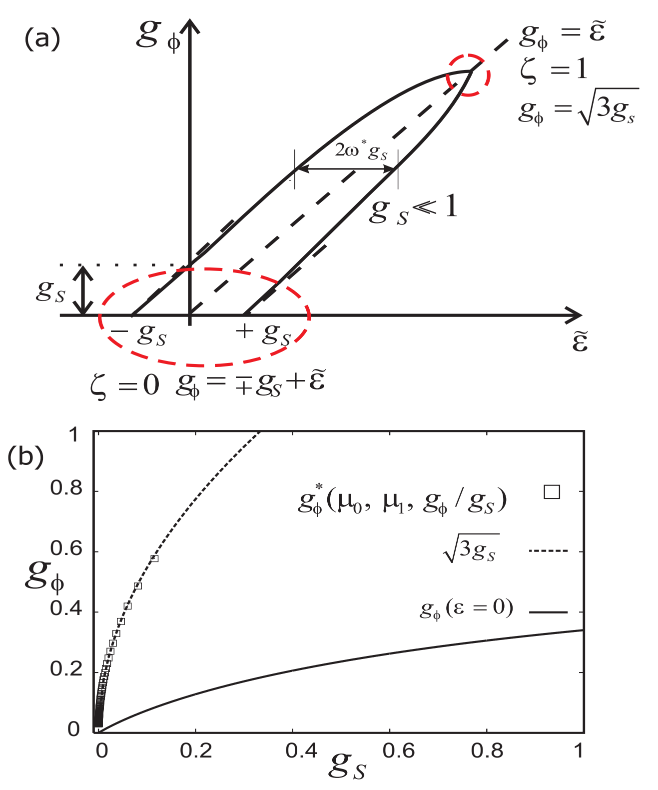

Fig. 3. (a) The sketch of the gap domain in $\tilde{\varepsilon} - g_\phi$ plane at $g_S \ll 1$. (b) Exact results (squares) for the maximum g_ϕ at $\zeta = 1$, follow the expected $\sqrt{3g_S}$ dependence for small g_S. The lower curve is the one plotted in Figure 2.

6 Gap-no gap boundary in general case

So far we have studied the boundary separating gap and no-gap solutions in the parameter space for two limiting cases that allow for analytic solutions. In this section, we find this boundary for arbitrary values of the parameters. We do this by solving equations (12–14) numerically.

Since we only concern with the boundary, the numerical procedure is as follows. We fix g_S and $\tilde{\varepsilon}$. The solutions of equation (12) with boundary condition (13) can be parametrized with μ_0. We express $\mu_1, \dot{\mu}_1$ in terms of μ_0. Then the last boundary condition (14) could be solved to find μ_0 in terms of g_ϕ. We do the opposite: We use (14) to directly express g_ϕ in terms of μ_0 and find the two extrema of $g_\phi(\mu_0)$. Those give the endpoints of the interval of g_ϕ where the gap solutions exist — elements of the boundary. We plot these endpoints at fixed g_S versus dimensionless energy $\tilde{\varepsilon}$ to obtain slanted strips similar to the one in Figure 3a. At certain energy, the extrema come together indicating the endpoint of the strip.

In Figure 4 we show these strips in $g_\phi - \tilde{\varepsilon}$ plane for a wide range of values of g_S. As expected from the previous discussion, for $g_S \ll 1$ the strips extend along the $g_\phi = \tilde{\varepsilon}$ line. The sharp tip of each strip gives the critical value of g_ϕ at which for a given g_S the induced minigap disappears. For small g_S, the height of the tip, $\sqrt{3g_S}$, is

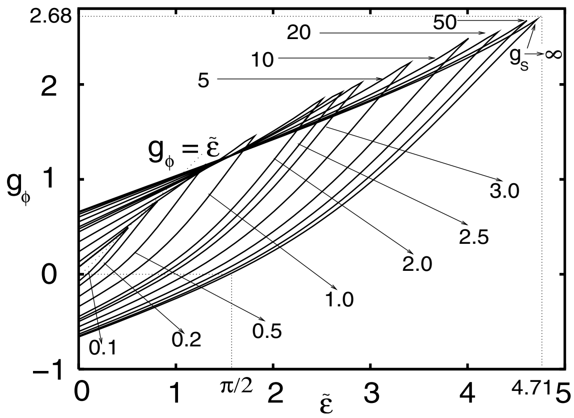

Fig. 4. The gap domains (strips) in $g_\phi - \tilde{\varepsilon}$ plane calculated for different values of $g_S = [0.1, 0.2, .., 50, \infty]$. Their shape is similar to the sketch in Figure 3a. The tip of each strip gives the maximum value of g_ϕ at which the gap survives and simultaneously the corresponding energy.

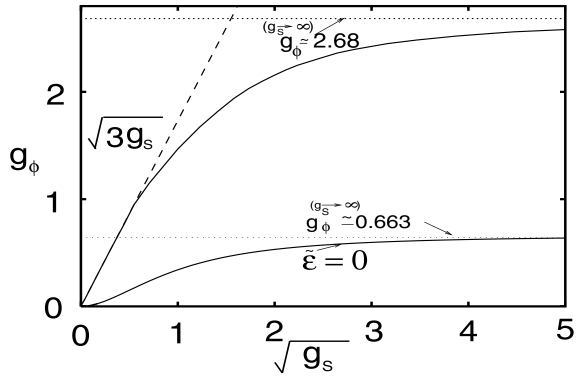

Fig. 5. Side view of the boundary surface in three-dimensional $(g_S, g_\phi, \tilde{\varepsilon})$ space. The upper curve gives the maximum g_ϕ connecting the tips of the strips plotted in Figure 4. It rises monotonically with g_S to reach the asymptotic value $\simeq 2.68$ at $g_S \to \infty$. The lower curve presents a cross-section of the surface with the $\tilde{\varepsilon} = 0$ plane.

much bigger than the width of the strip g_S. With increasing g_S, the shape of the strips changes. They increase both in width and height, so that these dimensions become of the same order. The strips also become less slanted. The shape converges at $g_S \to \infty$ (outer curve in Fig. 4). In this limit, the maximum g_ϕ that allows for superconductivity is $\approx$2.68 and is achieved at $\tilde{\varepsilon} \approx 4.71$. It is interesting to note that this energy is higher than the maximum value of the minigap without magnetic correlations ($\tilde{\varepsilon}(g_\phi = 0) = \pi/2$, see Fig. 4). Counterintuitevely, the presence of the magnetic insulator helps the gap solutions to persist at higher energy. Albeit the magnetic correlations quickly remove these solutions from the Fermi level.

Each strip is a cross-section of the boundary surface in three-dimensional $(g_\phi, g_S, \tilde{\varepsilon})$ space. We present in Figure 5 the side view of this surface. The lower curve in this figure is the cross-section of the surface with $\tilde{\varepsilon} = 0$ plane and shows the critical value of g_ϕ at which the gap solutions dissapears from the Fermi level. The same curve has been already presented in Figure 2. The upper curve gives the critical value of g_ϕ at which gap solutions dissapears at any energy. It consists of the tips of each strip from Figure 4 as a function of g_S. We see that at $g_S \to \infty$ this curve satures at $g_\phi = 2.68$. The asymptotics $g_\phi = \sqrt{3 g_S}$ derived in the previous section agree with this curve at $g_\phi < 1$ as expected.

7 Conclusions

We have studied the proximity effect in $S/N/FI$ structures with N being a diffusive normal metal. We pay special attention to the gap in the density of states and find its domain in the parameter space. The convenient dimensionless parameters to work with are g_ϕ, g_S that characterize the intensity of magnetic and superconducting correlations respectively, and energy in units of Thouless energy, $\tilde{\varepsilon}$.

We demonstrate that the combined effect of a ferromagnetic insulator and the elastic scattering on the proximity gap of a diffusive wire is twofold. First, the ferromagnetic insulator provides an effective exchange field $\tilde{h}$ that shifts the gap edges in opposite directions for opposite components without reducing the energy interval where the gap solutions occur. Second, its effect combined with sufficiently strong elastic scattering in N reduces this interval and finally suppresses the gap. In the limit of small g_S, g_ϕ ("spin-flip" limit) the mechanism of this suppression is precisely equivalent to the known one from magnetic impurities, with an effective spin-flip rate $1/\tau_s = g_\phi^2 E_T/3$. Qualitatively, this picture remains valid at arbitrary parameters.

If $g_\phi > 0.66$ no gap persist at the Fermi level. If $g_\phi > 2.68$ no gap occurs at any energy. Counterintuitevely, the gap in the presence of magnetic correlations may occur at energies higher than in the absence of the ferromagnetic insulator.

The absence or the presence of the gap in the normal part of a $S/N/FI$ structure at certain energy can be observed by a spin-sensitive tunnel probe. The resistance of such probe must exceed all interface resistances. The possible implementation of the probe depends on its concrete geometry. For traditional sandwich geometry, it is probably simpler to keep the FI layer rather thin, so that the electrons can tunnel through. Covering this layer with a conducting ferromagnet makes the tunnel probe. For the wire geometry, small tunnel contacts to ferromagnets can be made in different points of the normal wire. An alternative is the suggestion of reference [13]: the tunneling between two $FI/N/S$ structures.

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References

1. G. Deutscher, P.G. de Gennes, in *Superconductivity*, edited by R.D. Parks (Dekker, New York, 1969), p. 1005; P.G. de Gennes, Phys. Lett. **23**, 10 (1969); G. Deutscher, F. Meunier, Phys. Rev. Lett. **22**, 395 (1969); J.J. Hauser Phys. Rev. Lett. **23**, 374 (1969)
2. V.V. Ryazanov, V.A. Oboznov, A.Y. Rusanov, A.V. Veretennikov, A.A. Golubov, J. Aarts, Phys. Rev. Lett. **86**, 2427 (2001); T. Kontos, M. Aprili, J. Lesueur, F. Genet, B. Stephanidis, R. Boursier, Phys. Rev. Lett. **89**, 137007 (2002)
3. A.I. Buzdin, A.V. Vedyayev, N.V. Ryzhanova, Europhys. Lett. **48**, 686 (1999); L.R. Tagirov, Phys. Rev. Lett. 83, 2058 (1999)
4. A.F. Volkov, F.S. Bergeret, K.B. Efetov, Phys. Rev. Lett. **90**, 117006 (2003); F.S. Bergeret, A.F. Volkov, K.B. Efetov, Phys. Rev. Lett. **86**, 4096 (2001)
5. R. Mélin, S. Peysson, Phys. Rev. B **68**, 174515 (2003); R. Mélin, Eur. Phys. J. B **39**, 249 (2004); R. Mélin, D. Feinberg, `cond-mat/0407283`
6. G. Falci, D. Feinberg, F.W.J. Hekking, Europhys. Lett. **54**, 225 (2001); N.M. Chtchelkatchev, JETP Lett. **78**, 230 (2003)
7. P.M. Tedrow, R. Meservey, Phys. Rev. Lett. **27**, 919 (1971); R. Meservey, P.M. Tedrow, Phys. Rep. **238**, 173 (1994); J.Y. Gu, C.-Y. You, J.S. Jiang, J. Pearson, Ya.B. Bazaliy, S.D. Bader, Phys. Rev. Lett. **89**, 267001 (2002)
8. R. Fazio, C. Lucheroni, Europhys. Lett. **45**, 707 (1999); K. Halterman, O.T. Valls, Phys. Rev. B **65**, 014509 (2001); ibid. B **66**, 224516 (2002); B **69**, 014517 (2004); F.S. Bergeret, A.F. Volkov, K.B. Efetov, Phys. Rev. B **65**, 134505 (2002)
9. A.A. Abrikosov, L.P. Gorkov, Sov. Phys. JETP-USSR **12**, 1243 (1961)
10. M.J. DeWeert, G.B. Arnold, Phys. Rev. Lett. **55**, 1522 (1985); M.J. DeWeert, G.B. Arnold, Phys. Rev. B **39**, 11307 (1989)
11. T. Tokuyasu, J.A. Sauls, D. Rainer, Phys. Rev. B **38**, 8823 (1988)
12. P.M. Tedrow, J.E. Tkaczyk, A. Kumar, Phys. Rev. Lett. **56**, 1746 (1986)
13. D. Huertas-Hernando, Yu.V. Nazarov, W. Belzig, Phys. Rev. Lett. **88**, 047003 (2002)
14. A.F. Andreev, Sov. Phys. JETP **19**, 1228 (1964)
15. M. Tinkham, *Introduction to Superconductivity*, 2nd edn. (McGraw Hill, N.Y., 1996)
16. I.O. Kulik, Sov. Phys. JETP **30**, 944 (1970)
17. W.L. McMillan, Phys. Rev. **175**, 537 (1968)
18. S. Guéron, H. Pothier, Norman O. Birge, D. Esteve, M.H. Devoret, Phys. Rev. Lett. **77**, 3025(1996); W. Belzig, C. Bruder, G. Schön, Phys. Rev. B **54**, 9443 (1996); E. Scheer, W. Belzig, Y. Naveh, M.H. Devoret, D. Esteve, C. Urbina, Phys. Rev. Lett. **86**, 284 (2001); N. Moussy, H. Courtois, B. Pannetier, Europhys. Lett. **55**, 861 (2001)
19. A.A. Golubov, M.Yu. Kupriyanov, Sov Phys. JETP **69**, 805 (1989); A. Lodder, Yu.V. Nazarov, Phys. Rev. B **58**, 5783 (1998); S. Pilgram, W. Belzig, C. Bruder, Phys. Rev. B **62**, 12462 (2000); P.M. Ostrovsky, M.A. Skvortsov, M.V. Feigel'man, Phys. Rev. Lett. **87**, 027002 (2001)
20. K.D. Usadel, Phys. Rev. Lett. **25**, 507 (1970)
21. A.V. Zaitsev, Sov. Phys. JETP **59**, 1015 (1984)
22. A. Millis, D. Rainer, J.A. Sauls, Phys. Rev. B **38**, 4504 (1988)
23. Yu.V. Nazarov, Phys. Rev. Lett. **73**, 1420 (1994); Yu.V. Nazarov, Superlatt. and Microstruc. **25**, 1221 (1999)
24. J. Kopu, M. Eschrig, J.C. Cuevas, M. Fogelstrom, Phys. Rev. B **69**, 094501 (2004)
25. M. Zareyan, W. Belzig, Yu.V. Nazarov, Phys. Rev. Lett. **86**, 308 (2001)
26. J. Rammer, H. Smith, Rev. Mod. Phys. **58**, 323 (1986)
27. D. Huertas-Hernando, W. Belzig, Yu.V. Nazarov, `cond-mat/0204116`
28. A. Brataas, Yu.V. Nazarov, G.E.W. Bauer, Phys. Rev. Lett. **84**, 2481 (2000)
29. M. Eschrig, J. Kopu, J.C. Cuevas, G. Schön, Phys. Rev. Lett. **90**, 137003 (2003)
30. K. Maki *Gapless Superconductivity*, Chap. 18, Vol. 2 in *Superconductivity*, edited by R.D. Parks (Marcel Dekker, New York, 1969)